

Stochastic Model for a Vortex Depinning in Random Media

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We present a self-organized stochastic model for the dynamics of a single flux line in random media. The dynamics for the flux line in the longitudinal and the transversal direction to an averaged moving direction are coupled to each other. The roughness exponents of the flux line are measured for each direction, which are close to $\alpha_{\parallel} \approx 0.63$ for the longitudinal and $\alpha_{\perp} \approx 0.5$ for the transversal direction, respectively. The dynamic exponents are obtained as $z \approx 1$ for both directions. We discuss the classification of universality for the stochastic model.

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In the past few years, there has been an explosion of studies in the field of dynamics of fluctuating interfaces due to theoretical interests in the classification of universality for stochastic models and also due to applications to various physical phenomena such as crystal growth, vapor deposition, electroplating, biological growth, *etc.* A number of discrete models and continuum equations for interface dynamics have been introduced and studied [1-3]. An interesting feature of nonequilibrium interface dynamics is the nontrivial dynamic scaling behavior [4] of the interface fluctuation width, i.e.,

$$W(L, t) = \left\langle \frac{1}{L^{d'}} \sum_x (h(x, t) - \bar{h}(t))^2 \right\rangle^{1/2} \sim L^{\alpha} f(t/L^z), \quad (1)$$

where $h(x, t)$ is the height of site x on substrate at time t . \bar{h} , L , and d' denote the mean height, system size, and substrate dimension, respectively. The angular brackets stand for statistical average. The scaling function behaves as $f(x) \rightarrow \text{const}$ for $x \gg 1$, and $f(x) \sim x^{\beta}$ for $x \ll 1$ with $z = \alpha/\beta$. The exponents α , β and z are called the roughness, the growth, and the dynamic exponents, respectively.

Recently the problem of the pinning-depinning (PD) transition of interfaces in random media has also attracted interests in association with the problem of dynamics of fluctuating interfaces. Examples include the dynamics of domain boundaries of random Ising spin systems after being quenched below the critical temperature [6], wetting immiscible displacement of one fluid by another in a porous medium [5], pinning flux lines in type-II superconductors [7], fluid imbibition in paper [8], *etc.* In the problem of the PD transition, interface is pinned when external driving force F is weaker than pinning strength induced by random media, while it moves with a certain velocity v when the force F is greater than the pinning strength. Thus there exists a threshold of external applying force F_c across which the PD transition occurs. The role of the order parameter is played by the mean velocity, $v = \langle \sum_x \partial h(x, t) / \partial t \rangle / L^{d'}$. Accordingly,

the velocity is zero for $F < F_c$, and increases for $F > F_c$ as

$$v \sim (F - F_c)^{\theta}, \quad (2)$$

where the exponent θ is called the velocity exponent.

The continuum equation for the dynamics of interfaces in random media may be written simply as [9]

$$\frac{\partial h(x, t)}{\partial t} = \nu \nabla^2 h + F + \eta(x, h), \quad (3)$$

where $h(x, t)$ is the height of the interface at position x at time t . The first term on the right-hand side is from the smoothening effect of surface tension, the second term the uniform driving force, and the third a random force with short range correlations, satisfying $\langle \eta(x, h) \rangle = 0$ and $\langle \eta(x, h) \eta(x', h') \rangle = 2D \delta(x - x') \delta(h - h')$ with noise strength D . The above equation, called the quenched Edwards-Wilkinson (QEW) equation, would be relevant to the dynamics of the domain wall in random magnetic systems. More generally, recently a new continuum equation was introduced [10], which includes a nonlinear term $\frac{\lambda}{2} (\nabla h)^2$ induced from the anisotropic property of the pinning strength. Thus the equation is replaced by

$$\frac{\partial h(x, t)}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + F + \eta(x, h), \quad (4)$$

which is called the quenched Kardar Parisi Zhang (QKPZ) equation. The QKPZ equation leads to a different universality class from the QEW equation. Recently several stochastic models in the QKPZ universality class have been introduced [9,11]. From the models, it has been naturally concluded that the surface at the threshold of the PD transition F_c can be described by the directed percolation (DP) cluster spanned perpendicularly to the surface growth direction in 1+1 dimensions. The roughness exponent α of the interface is given as the ratio of the correlation length exponents ν_{\perp} and ν_{\parallel} of the DP cluster in the transversal and the longitudinal direction that is $\alpha = \nu_{\perp} / \nu_{\parallel} \approx 0.63$.

The dynamics of a single flux line in type-II superconductor with random impurities can also be understood

in a similar manner used in the dynamics of surface growth. The main difference between them lies in that the flux line is an one-dimensional chain embedded in three dimensions rather than in two dimensions. Thus the roughness of the flux line is quantified in two different directions, the longitudinal and the transversal to the direction of its averaged velocity. Recently the continuum equations for the flux line dynamics in each directions were derived by Ertas and Kardar [7], which are coupled each other and look very complicated in general. They obtained the roughness and the dynamic exponents for various cases of the coupled equations; however, there still remain several cases that the roughness and dynamic exponents are not determined. In this paper, we will introduce a simple self-organized stochastic model, which we believe is relevant to the dynamics of the flux line. The numerical results we obtain from the stochastic model will compensate for the roughness and the dynamic exponents for the unknown cases.

The stochastic model we introduce in this paper is defined as follows. First, we consider a body centered cubic (bcc) lattice, in which an elastic string runs along the x -direction as shown in Fig. 1. The discrete version of the elastic string is composed of L -massless beads (black dots) which locate at nearest neighboring sites of one another and they are connected with strings. The configuration looking a zig-zag as shown in Fig.1 is regarded as the initial flat configuration. The total length of the string is equal to $\sqrt{3}aL/2$, where a is a unit lattice constant of the bcc lattice and L is the total number of beads in the system. The elastic string does not run backwardly to the x -direction, so that (y, z) positions of the bead for each x are specified by single values. Each bead can be updated only in either positive y or positive z -directions according to the following rule. First, two random numbers are assigned on each bead, one of which is for positive y -direction and the other is for positive z -direction. The random numbers for the y -direction are uniformly distributed between $[0, p]$ where $p \leq 1$ and the ones for the z -direction are in $[0, 1]$. Next, a minimum random number is selected among the $2L$ random numbers of the entire system, through which we determine the bead and the direction to move. Then the bead having the minimum random number is updated by shifting its location by the lattice constant a along the direction chosen. Next, the avalanche process of updating may occur on neighboring beads when the separation between the nearest neighboring beads along the string becomes larger than $\sqrt{3}a/2$. In that case, the nearest neighboring bead is also shifted by a lattice constant along the direction already selected through the minimum random number. The avalanche rule is then applied successively to next neighboring beads to hold the conservation of the separation between nearest neighboring beads. The dynamic rule we used is similar to the Snejpen dynamic rule [12] but that the updating can occur in two different directions. Accordingly, we may say that our model is a

vector Snejpen model.

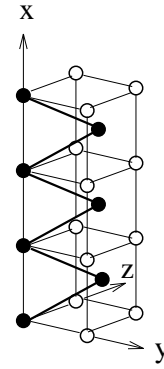


FIG. 1. The initial flat configuration of the discrete version of elastic string. Each bead has two-component noises representing random pinning forces in y and z directions.

The motion of the elastic string might be written by the following continuum equations,

$$\begin{aligned}\partial_t y(x, t) &= \nu_y \partial_x^2 y + \frac{\lambda_y}{2} (\partial_x y)^2 + \eta_y(x, y, z), \\ \partial_t z(x, t) &= \nu_z \partial_x^2 z + \frac{\lambda_z}{2} (\partial_x z)^2 + \eta_z(x, y, z),\end{aligned}\quad (5)$$

which are coupled via the quenched noise. The justification of the above equations associated with the stochastic model is based on that our model is similar to the Snejpen model but having the two different directions and the Snejpen model is a self-organized stochastic model belonging to the QKPZ universality in 1+1 dimensions. Even though the Snejpen model does not exhibit the PD transition, but the model gives the correct values of the roughness and the dynamic exponent in the QKPZ universality. Accordingly, we think that the self-organized stochastic model we introduced in this paper could give the correct values of the roughness and the dynamic exponents for the coupled QKPZ equations, Eq.(5).

When $p < 1$, the beads are more likely to advance in the y -direction, because random numbers are accumulated in the smaller interval $[0, p]$. The ratio of the averaged velocities v_z/v_y of the z and y directions is equal to p . Thus we can define the angle $\psi = \tan^{-1} p$ as the angle between the y -axis and the averaged propagating direction. Transforming the coordinate system (y, z) by the angle ψ into the new system $(r_{\parallel}, r_{\perp})$, the above equations can be rewritten in terms of r_{\parallel} and r_{\perp} , where r_{\parallel} is along the direction of the averaged velocity and r_{\perp} is perpendicular to r_{\parallel} . The continuum equations in the $(r_{\parallel}, r_{\perp})$ coordinate system are written as

$$\begin{aligned}\partial_t r_{\parallel}(x, t) &= \nu_{11} \partial_x^2 r_{\parallel} + \nu_{12} \partial_x^2 r_{\perp} + \frac{\lambda_{11}}{2} (\partial_x r_{\parallel})^2 + \\ &\quad \frac{\lambda_{12}}{2} (\partial_x r_{\perp})^2 + \frac{\lambda_{13}}{2} (\partial_x r_{\parallel})(\partial_x r_{\perp}) + \eta_{\parallel}(x, r_{\parallel}, r_{\perp}),\end{aligned}$$

$$\partial_t r_\perp(x, t) = \nu_{21} \partial_x^2 r_\parallel + \nu_{22} \partial_x^2 r_\perp + \frac{\lambda_{21}}{2} (\partial_x r_\perp)^2 + \quad (6)$$

$$\frac{\lambda_{22}}{2} (\partial_x r_\parallel)^2 + \frac{\lambda_{23}}{2} (\partial_x r_\perp)(\partial_x r_\parallel) + \eta_\perp(x, r_\parallel, r_\perp),$$

where ν_{ab} with $a, b \in 1$ or 2 are functions of $\nu_y, \nu_z, \cos \psi$, and $\sin \psi$, and λ_{ab} with $a, b \in 1, 2$, or 3 are of $\lambda_y, \lambda_z, \cos \psi$ and $\sin \psi$. η_\parallel and η_\perp are the random noises in the longitudinal and the transversal direction, respectively. When $p = 1$, the advance of the elastic string in y and z directions are identical, and the above equations are reduced simply as

$$\partial_t r_\parallel(x, t) = \nu_{11} \partial_x^2 r_\parallel + \frac{\lambda_{11}}{2} (\partial_x r_\parallel)^2 + \frac{\lambda_{12}}{2} (\partial_x r_\perp)^2 + \eta_\parallel, \quad (7)$$

$$\partial_t r_\perp(x, t) = \nu_{22} \partial_x^2 r_\perp + \frac{\lambda_{23}}{2} (\partial_x r_\perp)(\partial_x r_\parallel) + \eta_\perp. \quad (8)$$

Note that $\lambda_{11} \neq 0$ in Eq. (7) and Eq.(8) is invariant under $r_\perp \rightarrow -r_\perp$. In order to obtain the roughness exponents for each direction, we consider the spatial correlation functions C_\parallel and C_\perp after saturation,

$$C_\parallel(x, t) = \langle \frac{1}{L} \sum_{x_1} (r_\parallel(x + x_1, t) - r_\parallel(x_1, t))^2 \rangle^{1/2},$$

$$C_\perp(x, t) = \langle \frac{1}{L} \sum_{x_1} (r_\perp(x + x_1, t) - r_\perp(x_1, t))^2 \rangle^{1/2}, \quad (9)$$

which behave as $C_\parallel(x) \sim x^{\alpha_\parallel}$ and $C_\perp(x) \sim x^{\alpha_\perp}$. Next, in order to obtain the growth exponents, we consider the temporal correlation functions \tilde{C}_\parallel and \tilde{C}_\perp ,

$$\tilde{C}_\parallel(t_2 - t_1) = \langle \frac{1}{L} \sum_x (r_\parallel(x, t_2) - \bar{r}_\parallel(t_2) - r_\parallel(x, t_1) + \bar{r}_\parallel(t_1))^2 \rangle^{1/2},$$

$$\tilde{C}_\perp(t_2 - t_1) = \langle \frac{1}{L} \sum_x (r_\perp(x, t_2) - \bar{r}_\perp(t_2) - r_\perp(x, t_1) + \bar{r}_\perp(t_1))^2 \rangle^{1/2}, \quad (10)$$

where t_1 is taken as a saturation time. The correlation functions behave as $\tilde{C}_\parallel \sim (t_2 - t_1)^{\beta_\parallel}$ and $\tilde{C}_\perp \sim (t_2 - t_1)^{\beta_\perp}$. Numerical simulations are performed for the cases of $p = 1$ and $p = 1/2$.

For $p = 1$, where Eqs.(7-8) are hold, the roughness exponents are measured as $\alpha_\parallel \approx 0.63$ and $\alpha_\perp \approx 0.50$, and the data are shown in Fig.2. The growth exponents are measured as $\beta_\parallel \approx 0.64$ and $\beta_\perp \approx 0.53$, and the data are shown in Fig.3. From the numerical results, the dynamic exponents are obtained as $z_\parallel \approx 0.99$ and $z_\perp \approx 0.95$, which suggest that the true dynamic exponents be $z_\parallel = z_\perp = 1$. This result is attributed to that the coherent effect propagates along the string and the chemical distance between two points on string remains invariant through the process of the dynamics [13].

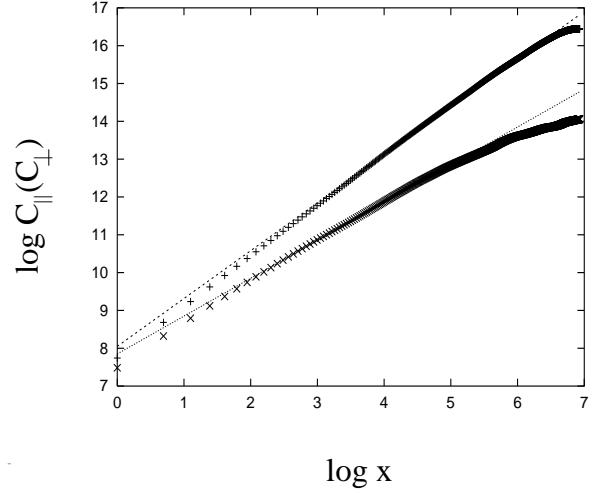


FIG. 2. Plot of C_\parallel (C_\perp) versus x for $p = 1$ in double logarithmic scales. The simulations are performed for the system size $L = 2048$. The lines obtained from the least square fits have the slopes 1.26 (top) and 1.00

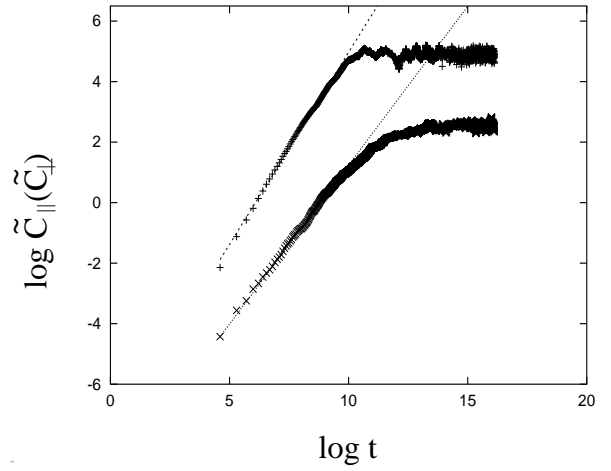


FIG. 3. Plot of \tilde{C}_\parallel (\tilde{C}_\perp) versus time for $p = 1$ in double logarithmic scales. The simulations are performed for the system size $L = 2048$. The lines obtained from the least square fit have the slopes 1.27 (top) and 1.05

Next for $p = 1/2$, the two spatial correlation functions for each direction are less distinctive than the case of $p = 1$ as shown in Fig.4. The roughness exponents are obtained as $\alpha_\parallel \approx 0.60$ and $\alpha_\perp \approx 0.54$. For the growth exponents, the time correlation function \tilde{C}_\parallel in the longitudinal direction exhibits the power-law dependent behavior against time with the exponent $\beta_\parallel \approx 0.64$. However, for the transversal direction, the data do not exhibit a simple power-law type behavior, rather they show a crossover behavior from $\beta_\perp \approx 0.50$ to $\beta_\perp < 0.50$ in Fig. 5. From the numerical results, it is likely that the roughness exponents are $\alpha_\parallel = 0.63$ and $\alpha_\perp = 0.5$, and the

dynamic exponents are $z_{\parallel} = z_{\perp} = 1$ even for $0 < p < 1$. Note that when $p = 0$, the dynamics in the transversal direction does not exist at all in our model. Accordingly, the numerical results indicate that the roughness and dynamic exponents do not depend on the angle ψ . Also the numerical values of the roughness and dynamic exponents suggest that the dynamics in the longitudinal direction belongs to the directed percolation depinning universality class (the QKPZ universality class) in 1+1 directions. On the other hand, for the transversal direction, the numerical values $\alpha_{\perp} \approx 0.5$ and $z_{\perp} \approx 1$ coincide with the ones in the anisotropic QKPZ universality class [10,14], even though the dynamics equation, Eq.(8), does not include the symmetric breaking term, $(\partial_x r_{\perp})^2$. We think that this coincidence is due to the presence of the term proportional to $\partial_x r_{\perp}$ in Eq.(8).

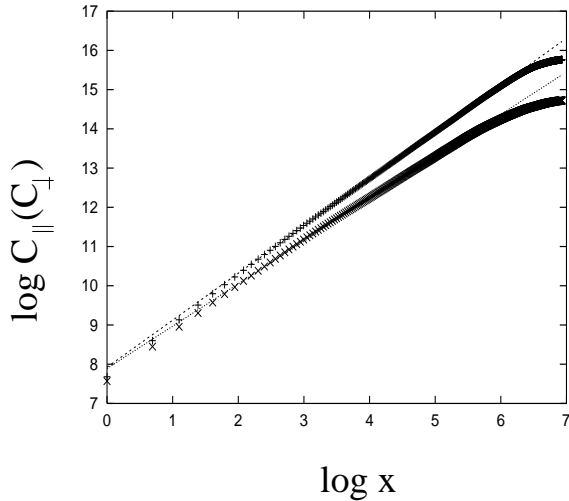


FIG. 4. Plot of C_{\parallel} (C_{\perp}) versus x for $p = 1/2$ in double logarithmic scales. The simulations are performed for the system size $L = 2048$. The lines obtained from the least square fit have the slopes 1.20 (top) and 1.08.

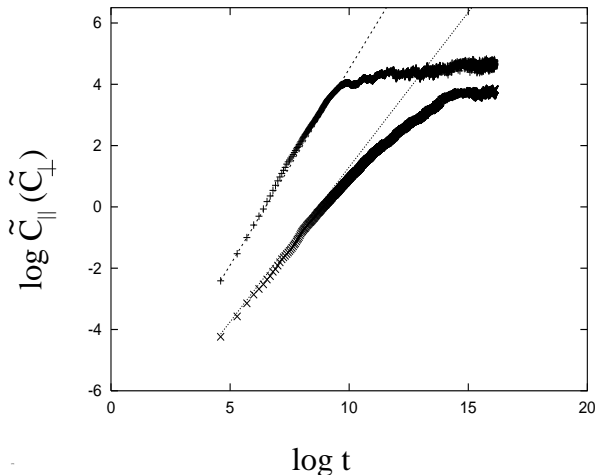


FIG. 5. Plot of \tilde{C}_{\parallel} (\tilde{C}_{\perp}) versus time for $p = 1/2$ in double logarithmic scales. The simulations are performed for the system size $L = 2048$. The lines have the slopes 1.28 (top) and 1.00

In summary, we have introduced a stochastic model associated with the dynamics of the flux line in quenched media at the depinning threshold. The conservation of the total length of string through the dynamics in the current stochastic model leads to $z = 1$ in both the longitudinal and the transversal direction. The roughness exponents were obtained as $\alpha_{\parallel} \approx 0.63$ and $\alpha_{\perp} \approx 0.5$, respectively.

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